Calculus – Lesson 5: Introduction to Limits (page 1)
Upward Bound Summer 2018 **Name: ____________________**

Basic: Limits at a Point

Sometimes a function approaches a value but never quite hits it. This is called a **limit**.

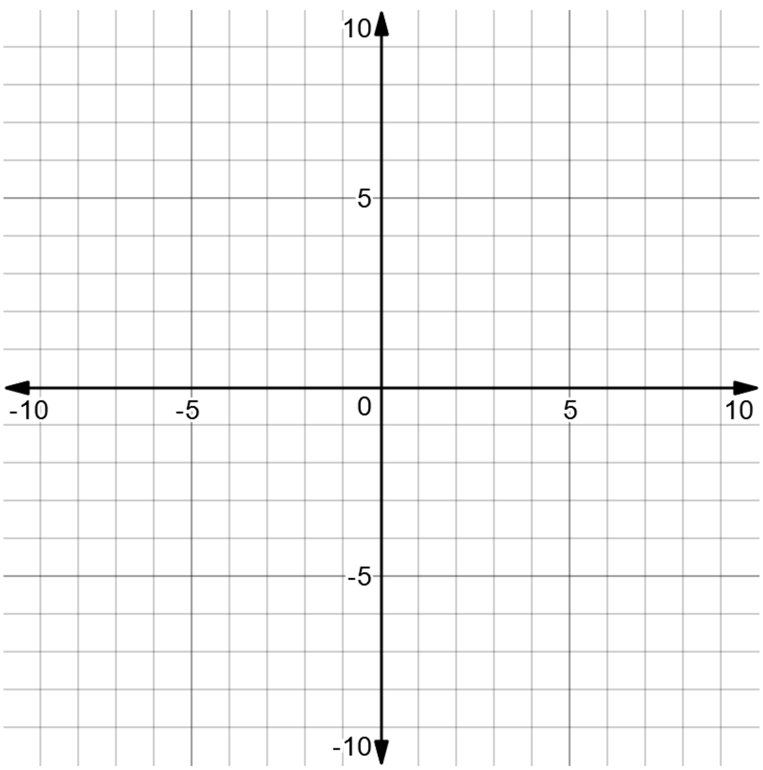
- Horizontal asymptotes indicate limits at infinity and/or negative infinity.
- Holes indicate limits at other values.

Informal Definition of a Limit:

Let f be a function. Then the limit of f at $x = a$ (denoted $\lim_{x \to a} f(x)$) is the value that the output of the function approaches as the input of the function approaches $x = a$.

(1) Define a rational function f that has a hole at $x = 4$.

(2) Graph your function f from Problem #1.



(3) What is $\lim_{x \to 4} f(x)$? Why?

(4) What is $\lim_{x \to 1} f(x)$? Why?

Calculus – Lesson 5: Introduction to Limits (page 2)
Upward Bound Summer 2018 **Name: ____________________**

When we talk about limits at a point, we usually mean **two-sided limits**. Whether you approach the x-value from the left or the right, you will be approaching the same y-value.

But we can also talk about **one-sided limits** if we use the proper notation.

- The limit from the left is denoted $\lim_{x \to a-} f(x)$.
- The limit from the right is denoted $\lim_{x \to a+} f(x)$.

A [two-sided] limit only exists if the limits from the left and right are equal.

For Problems #5-7, let f be the function defined by $f(x) = \frac{|x|}{x}$.

(5) What is $\lim_{x \to 0-} f(x)$?

(6) What is $\lim_{x \to 0+} f(x)$?

(7) Does $\lim_{x \to 0} f(x)$ exist? Why or why not?

Functions can sometimes be defined at a point and have a limit there, but they are not equal.

(8) Define a function g such that $\lim_{x \to -3} g(x)$ exists, -3 is in the domain of g, and $\lim_{x \to -3} g(x) \neq g(-3)$.
Explain how you know all conditions hold.

(9) Provide a definition for a function h such that $\lim_{x \to 2} h(x)$ does not exist, $\lim_{x \to 2-} h(x)$ exists, $\lim_{x \to 2+} h(x)$ exists, 2 is in the domain of h, and $\lim_{x \to 2-} h(x) \neq h(2) \neq \lim_{x \to 2+} h(x)$.
Explain how you know all conditions hold.

Calculus – Lesson 5: Introduction to Limits (page 3)
Upward Bound Summer 2018 **Name: ______________________**

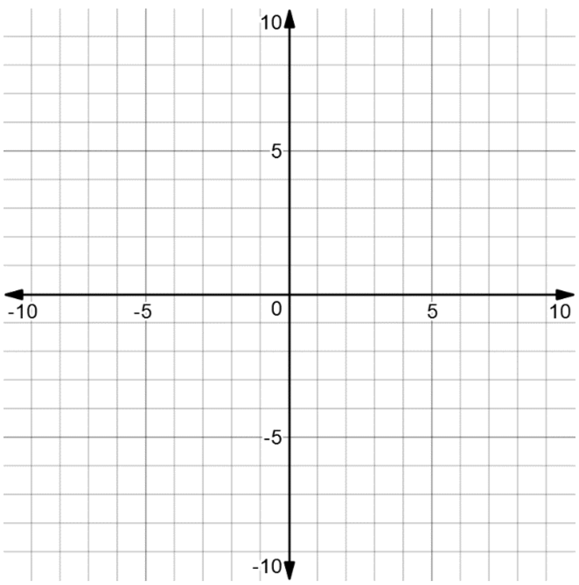
Intermediate: Limits at Infinity

The informal definition works for limits at infinity as well, if you allow the a to be infinity.

(10) Write in your own words an informal definition for a limit at infinity.

(11) Define a rational function f such that $\lim_{x\to\infty} f(x) = 3$.

(12) Graph your function f from Problem #11.

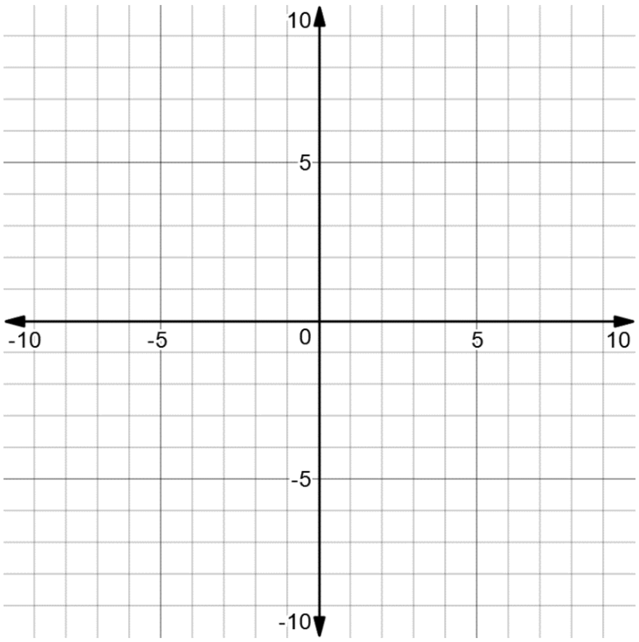


(13) What is $\lim_{x\to-\infty} f(x)$? Why?

(14) What is $\lim_{x\to 0} f(x)$? Why?

(15) Define a function g such that $\lim_{x\to\infty} g(x) = 3$ and g is *not* a rational function.

(16) Graph your function f from Problem #15.



(17) What is $\lim_{x \to -\infty} f(x)$? Why?

(18) Define a function h which has *two* distinct horizontal asymptotes. What are they?

<u>Advanced: Formal Definition of a Limit</u>

There is a more rigorous definition for a limit than the informal definition provided at the start of this lesson.

Formal Definition of a Limit:

Let f be a function defined on an interval containing $x = a$ (except possibly at $x = a$). Then the limit of f at $x = a$ is the value $\lim_{x \to a} f(x)$ such that for every $\varepsilon > 0$ there exists a $\delta > 0$ so that whenever $|x - a| < \delta$, $\left|f(x) - \lim_{x \to a} f(x)\right| < \varepsilon$.

(19) Let $f(x) = x^2$. Find a value $\delta > 0$ so that whenever $|x - 2| < \delta$, $|f(x) - 4| < 0.01$.

(20) Prove that $\lim_{x \to 2}(x^2) = 4$.